We discussed the goal of leveraged ETFs previously – to provide daily returns that match the desired ratio over the underlying index (https://www.cumber.com/margin-trading-vs-leveraged-etfs/). These ETFs rebalance daily to maintain the proportional leverage through derivatives such as futures, forwards, and swaps. We will demonstrate that this daily rebalancing feature dictates the long-term returns of leveraged ETFs, deviating from the multiple of the underlying index over the same period due to compounding.

One of our earlier commentaries compared the long-term returns of a leveraged ETF and an unleveraged index that suffers from a lack of compounding (http://www.cumber.com/leveraged-etfs/). We will revisit the issue with a simple example. If an index returned 30% in one year, then the arithmetic average daily return would be 0.1190%, using 252 trading days a year (1); however, the geometric average would be 0.1042% (2):

\[
30%/252 \approx 0.1190\% \quad (1)
\]
\[
(1+30\%)^{1/252} - 1 \approx 0.1042\% \quad (2)
\]

Apparently, requiring both daily and long-term returns of leveraged ETFs to match the underlying index is not realistic. Hence, given that leveraged ETFs’ target is to track the daily multiple returns, we recommend focusing on the daily tracking error.* We continue with our previous choice of the ETF SPXL as our example. We use one of the largest ETFs, SPY, as our comparison. First, we notice that the correlation between SPX and SPY has been lower than the correlation between SPX and SPXL (Table 1) since their inceptions. Moreover, the average daily tracking errors of SPY and SPXL are both very small, around 0.01%. The spread between the tracking errors is only 0.0034% (Table 2), net of expenses. On the other hand, we also
compare the absolute values of these daily tracking errors. Interestingly, even if both the absolute values are greater than before, the spread between the absolute averages is still relatively trivial – 0.0335%. From the daily return perspective, leveraged ETFs do not provide significantly higher tracking errors than their counterparts do.

Next, we will demonstrate that the compounding effect accounts mathematically for most of the long-term performance discrepancy. The approximation below shows that the compounded return of a leveraged ETF over a long period can deviate from the underlying index even without any tracking error.

\[
1 + R_{t,t+n} \approx (1 + \beta \bar{r})^n \exp \left[ -\frac{1}{2} \frac{\beta^2}{(1+\beta \bar{r})^2} nS^2 \right] \tag{3},
\]

where \( \beta \) is the leverage ratio, \( n \) is the number of holding days, \( \bar{r} \) is the index average return, \( S \) is the standard deviation of the daily returns, and \( R_{t,t+n} \) is the compounded return over the period, ignoring expenses. The main takeaway from equation (3) is that the higher the volatility \( S \) and the leverage ratio \( \beta \), the lower the compounded return \( R_{t,t+n} \) over the holding period. The return can be very poor in a sideways environment or rather pleasing in a bull market.

We can thus conclude that the so-called tracking error of a leveraged ETF over the long run is not really the same as the traditional tracking error; instead, it reflects the compounding effect. This effect is more pronounced with high leverage ratios and volatility. Alternatively, some traders
may refer to the effect as “time decay” or “volatility decay.” From the mathematical perspective, we can identify the compounding effect as the driving factor.

Leo Chen, Ph.D.
Portfolio Manager & Quantitative Strategist
Email | Bio

*We define daily tracking error as the difference between an ETF’s daily NAV return and the underlying index’s daily return.

**Data from Bloomberg.

***We adapt the derivation from the Richard Co and John Labuszewski paper “Leveraged ETFs: Where Is the Missing Performance?” (2009 ). We derive the approximation by applying Taylor series.

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